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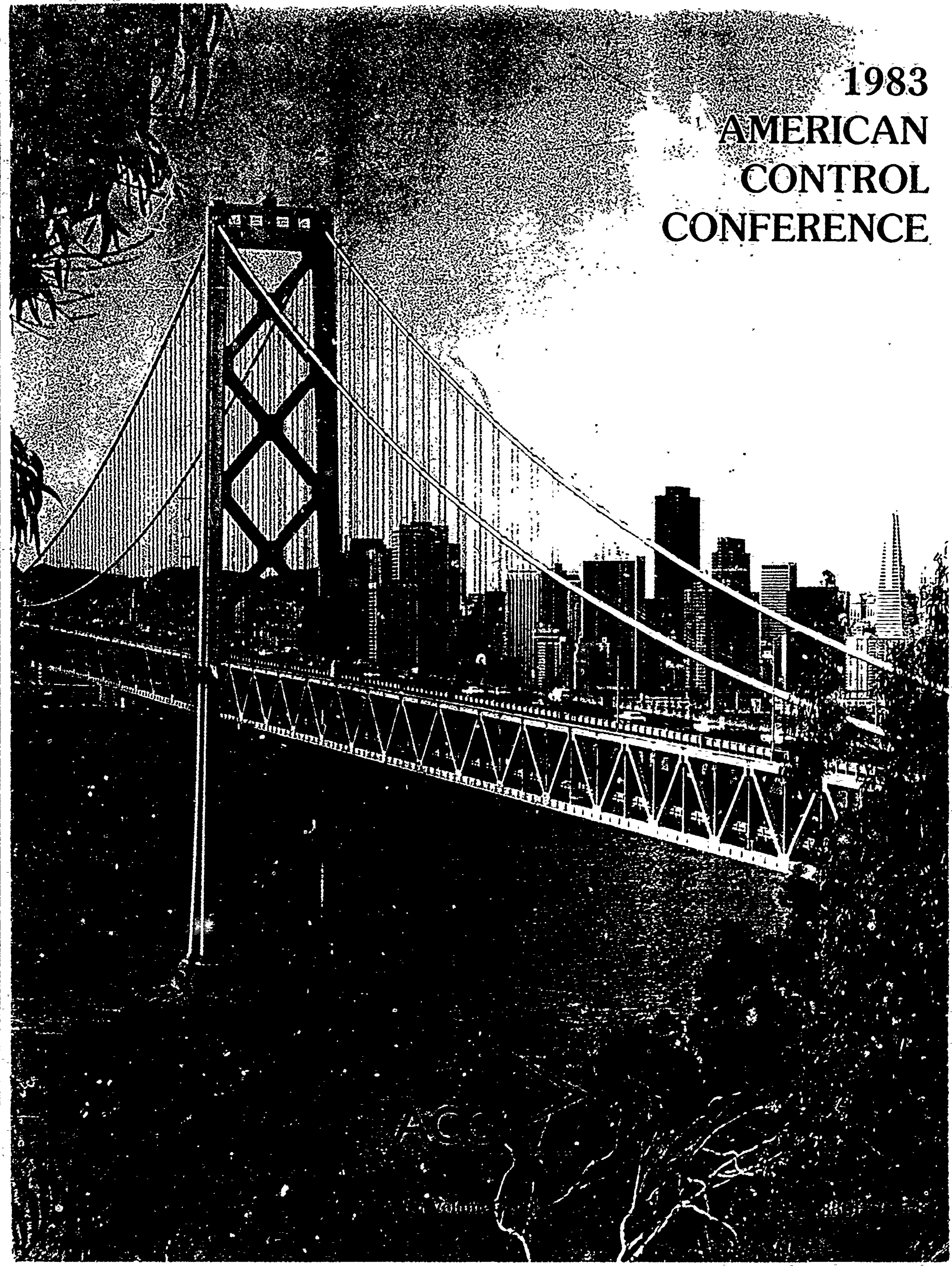

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APPLICATIONS OF FUZZY SET THEORY TO PARAMETER ESTIMATION AND TRACKING

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Summary

This paper develops a systematic approach to the estimation and diagnosis or classification of systems through the use of general fuzzy set systems. Some examples for tracking problems are illustrated.

Introduction

The paper presented here is a further development of previous work in the area of general diagnosis and estimation through use of general fuzzy set systems. (See, for example, [1-4] .) For purposes of clarification , many of the previously obtained results are extended and reformulated here.

The basic problem treated here consists of the following:

(1) An unknown state parameter vector consisting of subvectors whose values are known to lie in specified attribute domains. The vector is indexed by time and evolves according to a known process up to fuzzy errors, corresponding to prior possibility distributions which may be modeled empirically utilizing a panel of experts or obtained from physical and logical considerations.

(2) Observed data vectors arising from possibly many different sources and indexed also by time, corresponding to conditional data possibility distributions, modeled as in (1).

(3) An unknown diagnosis or classification parameter, corresponding to a possibility distribution over some known domain of values.

(4) A collection of fuzzy relations connecting the unknown state vector with the diagnosis parameter, often in the form of inference rules modeled as in (1).

Aspects of this problem have been treated in the literature from various viewpoints, but essentially using only Zadeh's original fuzzy set system, where negation is represented by 1-(.), conjunction by min, and disjunction by max. (See the comprehensive survey of Dubois and Prade [5], pp. 189-207 for dynamic fuzzy systems, and pp. 335-340 for fuzzy diagnosis. See also the interesting paper of Sira-Ramirez [6] concerning linear dynamic systems where measurement and state system errors only are modeled by fuzzy sets. He obtains a general solution for the fuzzy set of states, and, by specializing the fuzzy set error models to gaussian-like forms, obtains results similar to previous deterministic ellipsoidal bounded uncertainty approaches.)

First, general fuzzy set systems and their role in modeling natural language inputs are discussed. t-norms and t-conorms play a large role in this development. In turn, this leads to the concept of general fuzzy conditional sets, which leads to a general fuzzy Bayes' theorem. An application of the latter enables one to compute more complicated joint and posterior possibility distributions analagous to classical probability and Zadeh's fuzzy set approaches. In a different direction, it is shown that a feasible form may be obtained for the single best fuzzy set description of an unknown parameter through different sources. The main application of this result is that several possibility distributions describing the same object arising from different origins may be combined into a single possibility distribution by generalized conjunction and that this distribution maximizes the utilization of information present concerning the unknown object. Combining the last result with the application of fuzzy Bayes' theorem leads to the main result of the paper: a fuzzy set description of the state and diagnosis parameters based upon all the available information present-i.e., the observed data and relations or inference rules used. Finally, this result is specialized to the large class of Frankian fuzzy set systems, where a relatively simple approximation to the fuzzy maximum likelihood estimator of the state vector is obtained.

Analysis

Briefly (for a much more extensive discussion, see [7]), a <u>generalized fuzzy set system</u> F consists of the triple of operators (n,g,h) , representing negation, conjunction, and disjunction, respectively, relative to some fixed collection of spaces. These are augmented by arbitrary formations of cartesian products from which arbitrary strings of predicates of the form $< x\varepsilon A>$, interpreted as "x is in A" or as "x has attribute A", connected by the basic linguistic-multiple valued logic connectors "not", "and", "or" , are formed. Then choose n=1-(.) , g to be a <u>t-norm</u>, and h to be a <u>t-conorm</u>, with the last two connected usually by the DeMorgan relation, for all $x_j \varepsilon [0,1]$, $j=1,\ldots,n$,

$$h(x_1,\ldots,x_n) = 1-g(1-x_1,\ldots,x_n) \quad . \qquad (1)$$

(Because of their symmetry and associativity, t-norms and t-conorms may be defined as binary operators over $[0,1]^2$ and unambiguously extended to an arbitrary number of (at most countable, in general) of arguments. See [7,8] for a presentation of properties of t-norms and t-conorms.) The evaluation of the truth value of any of the above-mentioned strings can be accomplished by replacing the occurence of the linguistic-logical connector by the corresponding fuzzy set one with the interpretation for each elementary component predicate

$$\text{truth}(< x\varepsilon A>) = \phi_A(x) \quad , \qquad (2)$$

where $\phi_A(x)$ is the membership function of fuzzy subset A at x. Choice of which is the most appropriate

system for a given situation remains a difficult problem (see [9] for one approach) and Zadeh's original fuzzy set system $F=(1-(\cdot),\min,\max)$ may not always be suitable, in light of recent results connecting fuzzy set systems and random set ones (see, e.g., [9]).

General conditional fuzzy sets are defined simply from the relation

$$g(\phi_{A|y}(x), \phi_{\rho_Y(A)}(y)) = \phi_A(x,y) \quad , \qquad (3)$$

where $A|y$ is the fuzzy set from A conditioned on y and $\rho_Y(A)$ is the projection of A into Y given by

$$\phi_{\rho_Y(A)}(y) = \underset{(\text{over all } x\in X)}{h}(\phi_A(x,y)) \quad , \qquad (4)$$

where A is a fuzzy subset of $X\times Y$, for all $x\in X$ and $y\in Y$. (See [7] for additional details.) With this, fuzzy Bayes' theorem may now be presented:

Theorem 1. Fuzzy Bayes' Theorem

Fix fuzzy set system $F=(n, g, h)$; let B be a fuzzy subset of X and for each $x\in X$; let C_x be a fuzzy subset of Y. Then there exists a unique fuzzy subset A of $X\times Y$ such that

$$\rho_X(A) = B \quad , \qquad A|x = C_x \quad , \text{ all } x\in X \quad , \qquad (5)$$

$$\phi_A(x,y) = g(\phi_B(x), \phi_{C_x}(y)) \quad , \qquad (6)$$

in turn determining $\rho_Y(A)$ as in eq.(4), and finally determining $A|y$ by use of eq.(3) ∎

$\phi_{\rho_X(A)}$ is called the prior possibility distribution function for x. $\phi_{A|x}$ is called the conditional data possibility function for y given x, ϕ_A is called the joint possibility distribution function for x and y, and $\phi_{A|y}$ is called the posterior possibility distribution function for x given y. Thus one can formally identify all of the above with classical probability counterparts in Bayesian analysis, and for convenience, from now on, we will use the typical notation $p(x|y)$ for $\phi_{A|y}(x)$, $p(x,y)$ for $\phi_A(x,y)$, $p(y)$ for $\phi_{\rho_Y(A)}(y)$, etc.

Theorem 2. Application of Fuzzy Bayes' Theorem

Fix fuzzy set system F and suppose that

$$p(Q|\theta,y) = p(Q|\theta) \; ; \text{ all } Q,\theta,y \; . \qquad (7)$$

Then

$$p(Q,\theta|y) = g(p(Q|\theta), p(\theta|y)) \; , \qquad (8)$$

and hence

$$p(Q|y) = \underset{(\text{all } \theta)}{h}(g(p(Q|\theta), p(\theta|y))) \; . \qquad (9)$$

Note that although ∎

$$p(Q|y) \neq p(Q|\theta=y) \quad , \qquad (10)$$

when θ and y have the same range of values, because of the difference in meanings - θ- conditioning represents true values, while y-conditioning represents observed values, etc., nevertheless $p(Q|\theta=y)$ is a simple estimator of $p(Q|y)$, avoiding the possible tedious repetitious use of operator h in eq.(9).

Theorem 3. Uniformly Most Accurate Estimator

Let g be a fixed nondecreasing function-such as a t-norm- over $[0,1]^n$ - and suppose unknown θ is described by n possibility functions $p_1,\dots,p_n$ from different sources at confidence levels $\alpha_1,\dots,\alpha_n$, respectively:

$$p_j(\theta) \geq \alpha_j \; , \; j=1,\dots,n \quad \text{iff} \quad \theta\in \bigcap_{1\leq j\leq n} p_j^{-1}([\alpha_j,1]) \; . \qquad (11)$$

Then there exists a uniformly most accurate (see [3] for details) single fuzzy set description of θ at $g(\alpha_1,\dots,\alpha_n)$ confidence level, for all possible α_j's :

$$p(\theta) \overset{d}{=} g(p_1(\theta),\dots,p_n(\theta)) \geq g(\alpha_1,\dots,\alpha_n) \; . \qquad (12)$$

∎

Theorem 4. Basic Estimation-Classification Result

Fix fuzzy set system F; define $\theta^{(j)} \overset{d}{=} (\theta_0,\dots,\theta_j)$, $y^{(j)}$, etc., true attribute state vector $\theta_j \overset{d}{=} (\theta_{1j},\dots,\theta_{mj}) \in X_1\times\cdots\times X_m = X$; observed data vector $y_j\in Y$, X,Y countable; true classification parameter $Q\in Z$. For all $j\geq 1$, suppose $f_j: X\to X$, $k_j: Y\to Y$, $S_j: X\times X\to[0,1]$, $T_j: Y\times Y\to[0,1]$ and all $1\leq v\leq m$, $\sigma_v \overset{d}{=} (\sigma_{1v},\dots,\sigma_{mv}) \in X$, $W_{tv}: X_t\times X_t\to[0,1]$, $1\leq t\leq m$, $C_v: Z\to[0,1]$, all known, such that

$$p(\theta_j|\theta^{(j-1)}) = S_j(\theta_j, f_j(\theta_{j-1})) \; ; p(y_j|\theta_j) = T_j(y_j, k_j(\theta_j)) \; , \qquad (13)$$

$$p_v(Q|\theta', y^{(j)}) = p_v(Q|\theta') = \psi\big(\underset{1\leq t\leq m}{g}(W_{tv}(\theta'_t,\sigma_{tv})), C_v(Q)\big) \; , \qquad (14)$$

with inference rule p_v connecting any $Q\in Z$ and $\theta' \overset{d}{=} (\theta'_1,\dots,\theta'_m)\in X$, for $1\leq v\leq n$.

Then the uniformly most accurate single fuzzy set description, through $y^{(j)}$, of Q and θ_j is given in eqs.(8) and (9), where θ is replaced by θ_j, y by $y^{(j)}$, and where

$$p(Q|\theta) = \underset{(1\leq v\leq n)}{g}(p_v(Q|\theta_j)) \; ; \; p(\theta_j|y^{(j)}) = \underset{(\text{all } \theta^{(j-1)})}{h}(p(\theta^{(j)}|y^{(j)})) \; ; \qquad (15)$$

$$p(y^{(j)}|\theta^{(j)}) = \underset{(0\leq j'\leq j)}{g}(p(y_{j'}|\theta_{j'})) \; ; p(\theta^{(j)}) = \underset{(1\leq j'\leq j)}{g}(p(\theta_{j'}|\theta_{j'-1})) \; ; \qquad (16)$$

$$p(y^{(j)},\theta^{(j)}) = g(p(y^{(j)}|\theta^{(j)}), p(\theta^{(j)})) \; ; p(y^{(j)}) = \underset{(\text{all } \theta^{(j)})}{h}(p(y^{(j)},\theta^{(j)})) \; , \qquad (17)$$

and $p(\theta^{(j)}|y^{(j)})$ is determined from the equation

$$p(y^{(j)},\theta^{(j)}) = g(p(\theta^{(j)}|y^{(j)}), p(y^{(j)})) \; , \qquad (18)$$

where implication operator ψ is determined from

$$\psi(a,b) \overset{d}{=} h(n(a),b) \; ; \text{ all } a,b\in[0,1] \; . \qquad (19)$$

∎

Next, define for any unknown parameter θ and data vector y, the possibilistic maximum likelihood estimator of θ through y as that value of θ for which $\max_\theta p(y,\theta)$ occurs, equivalently, for which $\max_\theta p(\theta|y)$ occurs: $\hat{\theta}(y)$.

In addition, consider the DeMorgan Frankian-Archimedian class of fuzzy set systems $F_s = (1-(\cdot), g_s, h_s)$, $0\leq s\leq+\infty$,

$$g_s(a,b) \overset{d}{=} \log_s(1+(s^a-1)(s^b-1)/(s-1)) \; , \text{ all } a,b\in[0,1] \; . \qquad (20)$$

When $s=0$, $F_0=(1-(\cdot),\min,\max)$ and $s=1$, $F_1=(1-(\cdot),\text{prod},\text{probsum})$, limiting cases. (See [7] for details.)

Suppose now the hypothesis of Theorem 4 holds with $F=F_s$ for some s. Thus, eq.(16) is applicable, noting the maximization simplifies to that for $\prod((s^{p(y_{j'}|\theta_{j'})}-1)\cdot(s^{p(\theta_{j'}|\theta_{j'-1})}-1))$ over $0\leq j'\leq j$, if $s\neq 0,1$, or $+\infty$. (Corresponding forms hold for min, prod, and minbndsum. See [7].) Finally, a natural suboptimal estimator $\hat{\theta}(y^{(j)})$ relative to $\hat{\theta}$ can be obtained recursively from the equation (21)

$$g_s(y^{(j)},\hat{\theta}^{(j)}) = g_s\big(g_s(y^{(j-1)},\hat{\theta}^{(j-1)}), \max_{\theta_j}(g_s(p(y_j|\theta_j), p(\theta_j|\theta_{j-1}=\hat{\theta}_{j-1})))\big)$$

When $s=1$ and S_j and T_j are in translation gaussian-like form, $\hat{\theta}_j$ reduces to that estimator generated by the Kalman filter, except for the formal replacement everywhere of the state estimation covariances by the state noise covariances.

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